Physical and Monetary Input-Output Analysis:
What Makes the Difference?

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PHYSICAL AND MONETARY INPUT-OUTPUT ANALYSIS:

WHAT MAKES THE DIFFERENCE?

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Abstract

A recent paper in which embodied land appropriation of exports was calculated using a physical input-output model (Ecological Economics 44 (2003) 137-151) initiated a discussion in this journal concerning the conceptual differences between input-output models using a coefficient matrix based on physical input-output tables (PIOTs) in a single unit of mass and input-output models using a coefficient matrix based on monetary input-output tables (MIOTs) extended by a coefficient vector of physical factor inputs per unit of output. In this contribution we argue that the conceptual core of the discrepancies found when comparing outcomes obtained using physical vs. monetary input-output models lies in the assumptions regarding unit prices and not in the treatment of waste as has been claimed (Ecological Economics 48 (2004) 9-17). We first show that a basic static input-output model with the coefficient matrix derived from a monetary input-output table is equivalent to one where the coefficient matrix is derived from an input-output table in physical units provided that the assumption of unique sectoral prices is satisfied. We then illustrate that the input-output tables that were used in the original publication do not satisfy the assumption of homogenous sectoral prices, even after the inconsistent treatment of waste in the PIOT is corrected. We show that substantially different results from the physical and the monetary models in fact remain. Finally, we identify and discuss possible reasons for the observed differences in sectoral prices faced by different purchasing sectors and draw conclusions for the future development of applied physical input-output analysis.

Keywords: Physical input-output analysis, monetary input-output analysis, MIOT, PIOT, total physical resource requirements, sectoral unit prices

JEL Codes: C67, Q5
Introduction

Recently, a series of publications in this journal analyzed the use of input-output models based on coefficient matrices derived from input-output tables in mass units (PIOTs) for the computation of direct and indirect physical factor requirements to satisfy a given bill of final deliveries and compared them to input-output models using coefficient matrices derived from monetary tables (MIOTs) and a vector of physical factor inputs per unit of output.

Hubacek and Giljum (2003) claimed that physical input-output models are more appropriate to account for direct and indirect resource requirements (such as land area, raw materials, energy, or water). In their paper the authors computed direct and indirect land attributable to exports. They compared two models, one using a coefficient matrix based on a monetary table and the other using a coefficient matrix based on a physical table (both tables for Germany (Stahmer et al., 1998), highly aggregated to 3 x 3) and arrived at substantially different numerical results concerning overall and sectoral land appropriation for exports (see Hubacek and Giljum, 2003, table 3). The authors concluded: “For the quantification of direct and indirect resource requirements, we, therefore, suggest applying input-output analysis based on PIOTs, as (a) the most material-intensive sectors are also the sectors with the highest land appropriation and (b) physical input-output analysis illustrates land appropriation in relation to the material flows of each the sectors, which is more appropriate from the point of view of
environmental pressures than the land appropriation in relation to monetary flows of a MIOT” (Hubacek and Giljum, 2003, p 146).

In a reply to this paper Suh (2004) demonstrated that waste is misspecified in the PIOT used by Hubacek and Giljum. Suh showed that this misspecification makes the PIOT inconsistent in terms of mass balance and as a consequence is a flawed physical model. Furthermore, it violates the fundamental assumption of input-output economics that each sector produces a homogeneous characteristic output (Suh, 2004, pp 11). Suh proposed two alternative physical models (denoted as approach 1 and 2), which make use of PIOTs that consistently apply the mass balance assumptions and comply with standard assumptions of input-output analysis (Suh, 2004, table 3, p 14). Re-calculating Hubacek and Giljum’s original estimate of land appropriation of exports with the two alternative physical models, Suh arrived at quite different results from Hubacek and Giljum (Suh, 2004, fig.1, p15).

Suh determined that major differences in the results obtained with physical IO models and an extended monetary model are actually due to differences in the treatment of waste in the PIOT and not to a superiority of the physical model. He concludes: “These differences have nothing to do with the ‘resemblance with physical realities’ of PIOTs, as the results from a consistent PIOT approach may be similar to that of a MIOT approach. Nor does it prove the superiority of the physical table over the monetary
According to Suh, the differences between a monetary and a physical model may be relatively small if the treatment of waste is consistent in terms of mass balance and the underlying assumptions reasonably reflect real world conditions, two criteria which, according to Suh, apply to his approach 1.

Here we argue that the basic reason for both the large discrepancies found by Hubacek and Giljum (2003), and the smaller ones found by Suh (2004) when comparing physical and monetary input-output models, is that both the MIOT and the PIOT they used contradict the assumption of a unique price for the characteristic output of a given industry. We first show that a basic static input-output model using a coefficient matrix derived from a monetary input-output table is strictly equivalent to one where the coefficient matrix derived from an input-output table in physical units provided that the assumption of unique sectoral prices is satisfied. We then illustrate that neither the models (physical and monetary) used by Hubacek and Giljum (2003) nor the models proposed by Suh (2004) meet the assumption of homogenous sectoral prices and that in general single mass unit physical models deliver substantially different outcomes as compared to a monetary model for this reason. Finally, we identify and discuss possible reasons for the observed differences in sectoral prices and draw conclusions for the future development of applied physical input-output analysis.
The conceptual relation between a physical and a monetary model

The objective of this section is to demonstrate the equivalence between a basic input-output model with the variables measured in physical units and one with variables measured in money units on the assumption of a unique unit price for the characteristic output of each sector, as was shown in 1965 by Fisher who demonstrated that the Leontief system is not sensitive to a change of units ([Fisher 1965 15418 /id]).

To see this we start from the basic input-output equation:

\[ y = (I - A)x \quad (1) \]

where \( y \) is an \( n \times 1 \) vector of final deliveries and \( x \) is an \( n \times 1 \) vector of sectoral output, both measured in arbitrary physical units, in this case all output being measured in tons, and \( A \) is the \( n \times n \) matrix of coefficients of inputs per unit of output (tons per ton).

Let \( p \) be the \( n \times 1 \) vector of unit prices (price per ton). Pre-multiply both sides of (1) by \( \hat{p} \) and further pre-multiply \( x \) in the righthand side by \( I = [\hat{p} \quad \hat{p}^{-1}] \), where \( \hat{p} \) is the \( n \times n \)

\[ 1 \text{ We thank Sangwon Suh for making us aware of this reference.} \]
diagonal matrix with the price vector down the diagonal and $\hat{p}^{-1}$ is the n x n diagonal matrix with the inverse of the unit prices down the diagonal. This operation yields:

\[
\hat{p} y = \hat{p} (I - A) [\hat{p}^{-1} \hat{p}] x
\]

or

\[
\hat{p} y = [\hat{p} (I - A) \hat{p}^{-1}] \hat{p} x
\]

where $\hat{p} y$ is the n x 1 vector of final deliveries in monetary values, $\hat{p} (I - A) \hat{p}^{-1}$ is the n x n coefficient matrix in money values, and $\hat{p} x$ is the n x 1 vector of outputs in money values. Thus if (1) holds in physical units, then (2) holds in money units, and the converse is also easily demonstrated.

Note that if the coefficient matrices are derived from flow tables, the relationship between the table in physical units and that in monetary units can be formalized as follows. Let $Z^p$ be the n x n table of flows in physical units (in mixed units or, in this case, with all units in tons) and $Z^m$, the n x n table of flows in monetary values. Then an element of $Z^p$ is:
\{z_{ij}\} = \{a_{ij} x_j\}, \text{ where } x_j = \sum_j z_{ji} + y_j \tag{3}

and an element of $Z^m$ is:

\{z_{ij}^m\} = \{(p_j a_{ij} p_j^{-1}) (p_j x_j)\} = \{ p_j a_{ij} x_j\} \tag{4}

In other words, the element-by-element division of a column of the flow table in monetary units by the corresponding column in physical units must yield the n-vector $p$ of unit prices, \{p_j\}. The 2 models are the same except for the change of unit operation, and the vector of unit prices provides the information needed for the change of unit.

This proof holds in the case of a single physical unit, as in a PIOT, or in the more general case of mixed physical units.

It follows that a basic input-output model with the coefficient matrix based on a monetary input-output table conceptually must deliver exactly the same result as a basic input-output model where the coefficient matrix is based on a physical input-output table if the same assumptions are made about each sector’s factor requirements per unit of output.
The empirical differences found by Hubacek and Giljum (2003) and by Suh (2004)

We now turn to the empirical example provided by Hubacek and Giljum (2003) and discussed by Suh (2004). Suh showed that the treatment of waste in Hubacek and Giljum’s PIOT was inconsistent due to an inappropriate allocation of production waste in the PIOT because the wastes generated by each sector were added to its final deliveries in proportion to its use of primary inputs. As Suh rightly argued, the assumption of homogenous products is violated by combining production waste with commodity deliveries, and the mass balance principle is violated by allocating the production waste in proportion to primary inputs (Suh, 2004). We agree with this assessment but not with Suh’s claims that “the major differences in the results between the approaches are due mostly to the different principles in treating waste” and that “the pure difference between the PIOT [if handled consistently] and MIOT is relatively small” (Suh, 2004, p14). 2

Instead, we argue that the conceptual root of the differences in outcomes is explained by the fact that the two models are related by a price matrix rather than a price vector. That

2 The waste problem, important as it, is has been clearly privileged so far in the debate on physical and monetary input-output analysis. In contrast to this the role of prices although mentioned frequently (see Suh 2004, Giljum et al. 2004, Ditzenbacher 2004, and Giljum and Hubacek 2005) has not been fully recognized in its relevance.
is, different unit prices are being assumed for different purchasers of a given sector’s output. Our claim is demonstrated by a closer examination of the two tables.

INSERT TABLE 1 HERE

The price structure that relates the two input-output tables is readily derived by dividing each entry in the inter-industry and final demand tables of the MIOT by the corresponding entry of the PIOT (see Table 1). It is immediately evident that the implicit sectoral prices vary significantly. In the primary sector implicit prices vary by a factor of 13.6, in the secondary sector by a factor of 6, and in the tertiary sector by a factor of 30.7. Given these price assumptions, even a consistent treatment of waste can not assure comparable results from the two models. On the contrary, we must anticipate that the results from Suh’s (2004) physical model will differ substantially from the results of the monetary model.

Why then did Suh (2004), unlike Hubacek and Giljum (2003), arrive at similar although not identical results for total land requirements of exports using his physical model (approach 1) and the extended monetary model? We claim that the similarity between the two results gained by Suh is coincidental. This is illustrated in Figure 1, where we show the direct and indirect land appropriation gained from the monetary model and the physical model, not only for exports, as Suh did, but also for domestic final demand. We
used the same numerical figures as Hubacek and Giljum (2003) and the physical model (approach 1) proposed by Suh (2004).

**Insert Figure 1 here**

The results obtained by Suh (2004, p15) for exports are reproduced in the two lefthand bars in Figure 1. The two righthand bars show the corresponding results for domestic final demand. Differences in the composition of land requirements for domestic final demand are apparent between the physical model (Suh’s approach 1) and the monetary model. The apparent similarity between the land requirements for exports from the monetary and the physical models, visible in Figure 1, is simply a coincidence.

**Why are the unit prices of sectoral output different?**

We now bring together the conceptual and the empirical evidence to determine the assumptions needed for compatibility between the physical and monetary models. A basic assumption of input-output models is that sectoral prices are homogenous. From this it follows that it is a vector of prices that transforms an input-output model where the coefficient matrix is derived from a monetary input-output table to a corresponding model where the coefficient matrix is derived from an input-output table in physical units. When there is a single price vector, the two models are strictly equivalent. Under
these assumptions both models must deliver the same results for direct and indirect land requirements to satisfy a given final bill of goods.

In the empirical example, the physical model proposed by Suh (2004) delivers different results for the direct and indirect land requirements for final demand as compared to the extended monetary model. The MIOT and the PIOT used to compute the coefficient matrices of the two models are not related to each other by a price vector but rather by a price matrix, and this explains the difference between the results.

Next we inquire: Is it legitimate for the prices to be different? There are two possible responses. Either the construction of the PIOT is at fault, or else the standard input-output assumption of homogenous prices is too restrictive and may need to be challenged. We consider both options, starting with the construction of the PIOT.

*Construction of the physical table*

Only a few PIOTs have been compiled to date (Katterl and Kratena, 1990, Stahmer et al., 1997 and 1998, Stahmer, 2000, Pedersen, 1999, Mäenpää and Muukkonen, 2001, Statistischen Bundesamt, 2001, Nebbia, 2003). As has been pointed out before (see e.g. Strassert 2000) no standard methods for the compilation of PIOTs have yet been developed due to the relatively young history of physical input-output accounting. Therefore, it is highly likely that not only the factor input tables but also the inter-
industry and final demand tables of existing PIOTs differ in conventions and definitions both from each other and from monetary input-output tables, which have become much more standardized in the course of several decades of experience in compilation. This lack of standardisation for physical input-output accounting obstructs a reliable comparison between results obtained from a physical and a corresponding monetary input-output model as commensurability in the definition of the variables - a general requirement of any comparison - is not guaranteed.

In this section we investigate those construction principles of the German PIOT - the physical table which was used by Hubacek and Giljum in their original publication - that essentially affect the commensurability between the derived physical and the standard monetary model. We focus on elements other than the treatment of waste, as this aspect of the construction of the German PIOT (and of PIOTs in general) has already been discussed comprehensively by Suh (2004) and Dietzenbacher (2004).

It is crucial to emphasize that the entries in physical inter-industry and final demand tables will not necessarily correspond to commodity outputs. Instead they may be intended to measure a much more abstract concept of aggregate material flows from one sector to another. This is clearly the case in the physical inter-industry and final demand tables for Germany a 3 sector aggregation of the 12 sector inter-industry table of the German PIOT (Stahmer, 2000). Consequently, the ratio between a monetary flow and
the corresponding aggregate material flow does not measure the unit price of a commodity. It follows that there is no concept of commodity prices in the physical inter-industry and final demand tables for Germany. More specifically, the original German PIOT deviates in three important respects from the basic input-output concept of commodity flows.

First, the output of agriculture does not measure production of agricultural and forestry commodities but instead “corresponds to the total biomass increase of cultivated plants and animals (e.g., growth in woods cultivated as forestry operations, in cereals or potatoes, and the increase in livestock)” (Stahmer, 1998, p 16). The total biomass increase on agricultural land, managed forests and the total increase in livestock can hardly be interpreted as commodity output of the primary sector and therefore the derivation of a coefficient matrix from such PIOT will be grossly misleading for economic analysis.

The second issue concerns the service sectors. As Stahmer acknowledged in his later publications, measuring outputs of economic sectors in mass units has its clear limitation when it comes to sectors which normally have non-material outputs as is typically the case in the service sectors. He says: “Accounting schemes in mass units fall short when the results of economic processes are of a non-material nature, as e.g. services. In this extremely important area for economic analysis, additional information in monetary or
in time units is absolutely indispensable” (Stahmer, 2000, p 50, own translation). Nonetheless, the German PIOT measures the output of some of the service sectors in tons. As explained in the original publication of the German PIOT (Stahmer et al., 1997, and Stahmer et al., 1998), the outputs of the service sectors represent only a subset (selected on the basis of data availability) of some material flows associated with some services, such as e.g. “catering in restaurants,” a subset of the sector covering hotels and restaurants, homes and hostels, which involves deliveries of food and drinks to households. Consequently, the coefficient matrix derived from this PIOT is misleading as a representation of the inputs and outputs of this large and growing part of the economy. This fact explains the huge difference in the allocations of total land requirements to the service sectors between the monetary and the physical models (see Figure 1).

Finally, a third discrepancy in assumptions between the PIOT and standard input-output practices was introduced by Hubacek and Giljum (2003) and again in a later publication (Giljum and Hubacek, 2004), when they added the weight of waste to the weight of final demand, as pointed out in the third section of this paper.

Summarizing, the PIOT used by Hubacek and Giljum is not simply a conversion, by change of unit, of the monetary tables as Giljum et al. (2004) rightly observe in their reply to Suh (2004). The consequence, however, is that from such a PIOT one cannot
derive an input-output coefficient matrix that is commensurable to a coefficient matrix derived from a monetary input-output tables and hence no reliable comparison between a physical and the corresponding monetary input-output model can be based on it. As any input-output model makes use of a coefficient matrix which represents sectoral inputs per unit of total sectoral output, well-defined measurement of sectoral flows in the inter-industry table is essential for a meaningful interpretation of empirical results. A shared concept of the commodity-based characteristic output of a sector is indispensable for a meaningful comparison of results derived from input-output models using coefficient matrices based on monetary input-output tables and physical input-output tables, respectively.

*Which real-world conditions would justify different unit prices for a given commodity?*

There are two possible rationales for justifying different prices for the output of one sector when purchased by other sectors. The product mix delivered by sector i to sector j may vary significantly from the product mix delivered to sector k. Alternatively, even with the same product mix, the unit prices charged to different receiving sectors might be different.

The more detailed the sectoral classification, the narrower the range of commodities produced by a given sector: surely a three-sector aggregation is totally inadequate for drawing any empirical conclusions about product mixes. Part of the difference obtained
in the comparison made by Giljum and Hubacek (2003) may thus simply be attributed to the inappropriately high level of aggregation at which they carried out their analysis. Until now, most input-output studies have been based mainly on monetized tables with little if any use of explicit price vectors. Steps are now being taken toward integrated, but distinct, quantity and price analyses; see, an overview of this subject by Duchin (2005). Consequently, we can expect greater scrutiny of the role of unit prices, including the practice of assuming a common price for all purchasers. In any case, explicit and comparable price assumptions and some consensus about empirically meaningful levels of aggregation will need to be made for work with physical and monetized models.

Conclusions:

The conceptual core of the different results that arise from physical or extended monetary input-output models lies in the assumptions, explicit or implicit, about unit prices. The basic input-output model assumes homogenous sectoral prices for commodity outputs. We have shown that if there is a single price vector, the two models are equivalent except for the change of unit operation. Consequently, the calculation of physical amounts of factor inputs required to produce given final deliveries will be exactly the same if a physical model (be it in single or mixed units) is used or a monetary model extended by a vector of physical factor inputs. The discrepancies between the two solutions found by Hubacek and Giljum (2003), and also by Suh (2004)
and Dietzenbacher (2004) after correcting for the treatment of waste, are explained by the fact that the two tables on which the monetary and the physical models are based are not related to each other by a price vector, i.e., a single price for the characteristic output of each sector, but rather by a price matrix.

We identified three possible reasons why unit prices appear to be different when physical or monetary input output models are used.

First, an inappropriately high level of aggregation. The assumption of a single price for the characteristic output of a sector becomes more reliable the more detailed the resolution of the commodity classification. A 3 x 3 coefficient matrix, as used by Hubacek and Giljum (2003), is an inadequate basis for drawing empirical conclusions about differences between monetary and physical input output models. Empirically meaningful levels of aggregation are required for ecological input-output analysis.

Second is the concept behind the definition of commodity flows in the physical table. Since unit prices are defined per unit of commodity or service, an appropriate representation of unit prices relies on a reliable measure for the quantity of the flows of commodities (including services). The PIOT used by Hubacek and Giljum (2003) fails to provide reliable physical values to quantify the flows of commodities (including services) among the sectors of the economy even after the correction for the treatment of
waste. Therefore, the prices applied to these physical flows have no empirical significance.

Third, the assumption of unique sectoral prices is inappropriate. As we have shown, the input-output tables used by Hubacek and Giljum (2003) are insufficient to draw empirical conclusions regarding unit prices, i.e., if deliveries of different product mixes to different sectors or if different prices charged on the same product mix are of quantitative importance in real world economies. Conceptually, however, it should be noted that the assumption of a unique sectoral price is a requirement for a quantity model based on a monetary input-output table that incorporates this assumption. The desirability of dropping the assumption of homogenous sectoral prices is outside the scope of this paper.

Our conviction is that input-output coefficient matrices should be measured in mixed units, not in a single, aggregated mass unit nor in a single, aggregated monetary unit but rather the most appropriate unit for measuring the characteristic output of each sector.

The determination of physical quantities of factor inputs needed to deliver a given final bill of goods requires a coefficient matrix with appropriate physical units for the characteristic output of each sector. When constructed for the past, these matrices will be exactly equivalent to monetized matrices given the assumption of a homogeneous price
for each sector’s output. They will however convey additional information as they
distinguish physical quantities from unit prices. When projected coefficient matrices for
the future are required (these matrices are projected directly rather than being derived
from a flow table), they are more readily constructed in physical units rather than
monetary units, since the latter would require projections of future changes in relative
prices as well as future changes in technologies. These statements are true in the case of
the standard input-output assumption of a unique sectoral price. They also hold if one
wants to entertain the assumption of a matrix of prices.

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http://econpapers.hhs.se/paper/rpirpiwpe/0424.htm


Table 1: Implicit sectoral prices of commodity outputs in DM per ton, Germany 1990

<table>
<thead>
<tr>
<th>DM/ton</th>
<th>primary sector</th>
<th>secondary sector</th>
<th>tertiary sector</th>
<th>domestic final demand</th>
<th>exports</th>
</tr>
</thead>
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<td>primary sector</td>
<td>0.02</td>
<td>0.06</td>
<td>0.24</td>
<td>0.04</td>
<td>0.27</td>
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<tr>
<td>secondary sector</td>
<td>1.19</td>
<td>0.63</td>
<td>2.07</td>
<td>0.84</td>
<td>3.80</td>
</tr>
<tr>
<td>tertiary sector</td>
<td>5.39</td>
<td>5.31</td>
<td>45.71</td>
<td>163.09</td>
<td>5.67</td>
</tr>
</tbody>
</table>

*Source: calculated from aggregated MIOT and PIOT for Germany in 1990 (Stahmer 2000), as aggregated by Hubacek and Giljum (2003)*
Figure 1: Total (direct and indirect) land appropriation for exports and for domestic final demand